POWER SYSTEM ANALYSIS and STABILITY

Presentation By

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UNIT I

INTRODUCTION
Power system network

![Power system network diagram]

- GS 11kV
- 11kV/130kV
- 130kV/33kV
- 33kV/11kV
- 11kV/415V or 230V

Loads
SINGLE LINE DIAGRAM

It is a diagrammatic representation of a power system in which the components are represented by their symbols.
COMPONENTS OF A POWER SYSTEM

1. Alternator
2. Power transformer
3. Transmission lines
4. Substation transformer
5. Distribution transformer
6. Loads
MODELLING OF GENERATOR AND SYNCHRONOUS MOTOR

1Φ equivalent circuit of generator

1Φ equivalent circuit of synchronous motor
MODELLING OF TRANSFORMER

\[ K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} \]

\[ R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} \quad = \text{Equivalent resistance referred to } 1^\circ \]

\[ X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} \quad = \text{Equivalent reactance referred to } 1^\circ \]
MODELLING OF TRANSMISSION LINE

Π type

R

-jXc

-j2Xc

-j2Xc

T type

R/2

jX_L/2

R/2

jX_L/2

-jXc
MODELLING OF INDUCTION MOTOR

\[ R_r \left( \frac{1}{s} - 1 \right) \] = Resistance representing load

\[ R = R_s + R_r' \] = Equivalent resistance referred to stator

\[ X = X_s + X_r' \] = Equivalent reactance referred to stator
per unit = actual value/base value

Let $KVA_b =$ Base KVA

$kV_b =$ Base voltage

$Z_b =$ Base impedance in $\Omega$

$$Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(kV_b)^2}{KVA_b} = \frac{(kV_b)^2}{1000}$$
Changing the base of per unit quantities

Let $z =$ actual impedance ($\Omega$)

$Z_b =$ base impedance ($\Omega$)

Let $kV_{b,old}$ & $MVB_{b,old}$ represent old base values

$kV_{b,new}$ & $MVB_{b,new}$ represent new base values
\[ Z_{p.u,\text{old}} = \frac{Z \cdot MVA_{b,\text{old}}}{(kV_{b,\text{old}})^2} \rightarrow (1) \]

\[ Z = \frac{Z_{p.u,\text{old}} \cdot MVA_{b,\text{old}}}{(kV_{b,\text{old}})^2} \rightarrow (2) \]

\[ Z_{p.u,\text{new}} = \frac{Z \cdot MVA_{b,\text{new}}}{(kV_{b,\text{new}})^2} \rightarrow (3) \]

\[ Z_{p.u,\text{new}} = Z_{p.u,\text{old}} \cdot \frac{(kV_{b,\text{old}})^2}{(kV_{b,\text{new}})^2} \cdot \frac{MVA_{b,\text{new}}}{MVA_{b,\text{old}}} \]
ADVANTAGES OF PER UNIT CALCULATIONS

- The p.u impedance referred to either side of a 1Φ transformer is same.
- The manufacturers provide the impedance value in p.u.
- The p.u impedance referred to either side of a 3Φ transformer is same regardless of the 3Φ connections Y-Y,Δ-Y.
- p.u value always less than unity.
IMPEDANCE DIAGRAM

• This diagram obtained by replacing each component by their 1Φ equivalent circuit.

Following approximations are made to draw impedance diagram
1. The impedance b/w neutral and ground omitted.
2. Shunt branches of the transformer equivalent circuit neglected.
REACTANCE DIAGRAM

- It is the equivalent circuit of the power system in which the various components are represented by their respective equivalent circuit.

- Reactance diagram can be obtained after omitting all resistances & capacitances of the transmission line from impedance diagram.
REACTANCE DIAGRAM FOR THE GIVEN POWER SYSTEM NETWORK
PROCEDURE TO FORM REACTANCE DIAGRAM FROM SINGLE DIAGRAM

1. Select a base power kVA$_b$ or MVA$_b$
2. Select a base voltage kV$_b$
3. The voltage conversion is achieved by means of transformer kV$_b$ on LT section = kV$_b$ on HT section x LT voltage rating / HT voltage rating
4. When specified reactance of a component is in ohms
   p.u reactance = actual reactance / base reactance

   specified reactance of a component is in p.u

\[
X_{p.u,new} = X_{p.u,old} \times \frac{\left( kV_{b,old} \right)^2}{\left( kV_{b,new} \right)^2} \times \frac{MVA_{b,new}}{MVA_{b,old}}
\]
p.u. calculation of 3 winding transformer

\[ Z_p = \text{Impedance of primary winding} \]
\[ Z_s' = \text{Impedance of secondary winding} \]
\[ Z_t' = \text{Impedance of tertiary winding} \]

Short circuit test conducted to find out the above 3 impedances.
\[
Z_p = \frac{1}{2} \left[ Z_{ps} + Z_{pt} - Z_{st}' \right]
\]
\[
Z_s' = \frac{1}{2} \left[ Z_{ps} + Z_{st}' - Z_{pt} \right]
\]
\[
Z_t' = \frac{1}{2} \left[ -Z_{ps} + Z_{pt} + Z_{st}' \right]
\]

\(Z_{ps}\) = Leakage impedance measured in 1\(^o\) with 2\(^o\) short circuited and tertiary open.

\(Z_{pt}\) = Leakage impedance measured in 1\(^o\) with tertiary short circuited and 2\(^o\) open.

\(Z_{st}'\) = Leakage impedance measured in 2\(^o\) with tertiary short circuited and 1\(^o\) open and referred to primary
PRIMITIVE NETWORK

It is a set of unconnected elements which provides information regarding the characteristics of individual elements. It can be represented both in impedance & admittance form.

\[ e_{rs} + Z_{rs} = v_{rs} = E_r - E_s \]

Impedance form

\[ j_{rs} + i_{rs} = v_{rs} = E_r - E_s \]

Admittance form
BUS ADMITTANCE (Y BUS) MATRIX

Y BUS can be formed by 2 methods

1. Inspection method
2. Singular transformation

\[
Y_{\text{BUS}} = \begin{pmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1n} \\
Y_{21} & Y_{22} & \cdots & Y_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{n1} & Y_{n2} & \cdots & Y_{nn}
\end{pmatrix}
\]
INSPECTION METHOD

For n bus system

Diagonal element of Y BUS

\[ Y_{ii} = \sum_{j=1}^{n} y_{ij} \]

Off Diagonal element of Y BUS

\[ Y_{ij} = -y_{ij} \]
SINGULAR TRANSFORMATION METHOD

\[ Y \text{ BUS} = A^T[y]A \]

Where \([y]\)=primitive admittance

\[ A = \text{bus incidence matrix} \]
ALGORITHM FOR FORMATION OF THE BUS IMPEDANCE MATRIX

- Modification of Zbus matrix involves any one of the following 4 cases

Case 1: adding a branch impedance $z_b$ from a new bus $p$ to the reference bus
- Addition of new bus will increase the order the $Z_{bus}$ matrix by 1

$$Z_{bus,new} = \begin{pmatrix} z_{original} & 0 \\ 0 & z_b \end{pmatrix}$$

(n+1)th column and row elements are zero except the diagonal diagonal element is $z_b$
Case 2: adding a branch impedance $z_b$ from a new bus $p$ to the existing bus $q$

Addition of new bus will increase the order the $Z_{bus}$ matrix by 1

The elements of $(n+1)$th column and row are the elements of $q$th column and row and the diagonal element is $Z_{qq} + Z_b$

Case 3: adding a branch impedance $z_b$ from an existing bus $p$ to the reference bus

The elements of $(n+1)$th column and row are the elements of $q$th column and row and the diagonal element is $Z_{qq} + Z_b$ and $(n+1)$th row and column should be eliminated using the following formula

$$Z_{jk, act} = Z_{jk} - \frac{Z_j(n+1)Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \quad j = 1, 2...n; k = 1, 2..n$$
Case 4: adding a branch impedance $z_b$ between existing buses $h$ and $q$

Elements of $(n+1)$th column are elements of bus $h$ column – bus $q$ column and elements of $(n+1)$th row are elements of bus $h$ row – bus $q$ row the diagonal element = $Z_{b} + Z_{hh} + Z_{qq} - 2Z_{hq}$

and $(n+1)$th row and column should be eliminated using the following formula

$$Z_{jk,act} = Z_{jk} - \frac{Z_{j(n+1)}Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \quad j = 1, 2 \ldots n; k = 1, 2 \ldots n$$
UNIT II

POWER FLOW ANALYSIS
BUS CLASSIFICATION

1. Slack bus or Reference bus or Swing bus:
   $|V|$ and $\delta$ are specified. P and Q are unspecified, and to be calculated.

2. Generator bus or PV bus or Voltage controlled bus:
   P and $|V|$ are specified. Q and $\delta$ are unspecified, and to be calculated.

3. Load bus or PQ bus:
   P and Q are specified. $|V|$ and $\delta$ are unspecified, and to be calculated.
ITERATIVE METHOD

\[ I_p = \sum_{q=1}^{n} Y_{pq} V_q \]

\[ S_p = P_p - jQ_p = V_p^* I_p \]

\[ \frac{P_p - jQ_p}{V_P^*} = \sum_{q=1}^{n} Y_{pq} V_q \]

The above Load flow equations are non linear and can be solved by following iterative methods.

1. Gauss seidal method
2. Newton Raphson method
3. Fast Decoupled method
GAUSS SEIDAL METHOD

For load bus calculate $|V|$ and $\delta$ from $V_p^{k+1}$ equation

$$V_p^{k+1} = \frac{1}{Y_{pp}} \left[ \frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^{k} \right]$$

For generator bus calculate $Q$ from $Q_p^{K+1}$ equation

$$Q_p^{k+1} = -1 \cdot \text{Im} \left\{ (V_p^k)^* \left[ \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^{n} Y_{pq} V_q^{k} \right] \right\}$$
• Check $Q_{p,\text{cal}}^{k+1}$ with the limits of $Q_p$

• If $Q_{p,\text{cal}}^{k+1}$ lies within the limits bus $p$ remains as PV bus otherwise it will change to load bus

• Calculate $\delta$ for PV bus from $V_p^{k+1}$ equation

• Acceleration factor $\alpha$ can be used for faster convergence

• Calculate change in bus-$p$ voltage

\[
\Delta V_p^{k+1} = V_p^{k+1} - V_p^k
\]

• If $|\Delta V_{\text{max}}| < \varepsilon$, find slack bus power otherwise increase the iteration count (k)

• Slack bus power = $\sum S_G - \sum S_L$
NEWTON RAPHSON METHOD

\[ P_i - Q_i = \sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| [\theta_{ij} - \delta_i + \delta_j] \]

\[ P_i = \sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \]

\[ Q_i = \sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \]

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
J_1 & J_2 \\
J_3 & J_4
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta |V|
\end{bmatrix}
\]

\[ \Delta P_i^k = P_i^{sch} - P_i^k \]

\[ \Delta Q_i^k = Q_i^{sch} - Q_i^k \]
• Calculate $|V|$ and $\delta$ from the following equation

$$\delta_{i}^{k+1} = \delta_{i}^{k} + \Delta \delta^{k}$$

$$|V_{i}^{k+1}| = |V_{i}^{k}| + \Delta |V_{i}^{k}|$$

• If

$$\Delta P_{i}^{k} < \varepsilon$$

$$\Delta Q_{i}^{k} < \varepsilon$$

• stop the iteration otherwise increase the iteration count (k)
FAST DECOUPLED METHOD

- \( J_2 \) & \( J_3 \) of Jacobian matrix are zero

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
J_1 & 0 \\
0 & J_4
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta |V|
\end{bmatrix}
\]

\[
\Delta P = J_1 \Delta \delta = \left[ \frac{\partial P}{\partial \delta} \right] \Delta \delta
\]

\[
\Delta Q = J_4 \Delta |V| = \left[ \frac{\partial Q}{\partial |V|} \right] \Delta |V|
\]

\[
\frac{\Delta P}{\Delta |V_i|} = -B' \Delta \delta
\]

\[
\frac{\Delta Q}{\Delta |V_i|} = -B'' \Delta |V|
\]

\[
\Delta \delta = -\left[ B' \right]^{-1} \frac{\Delta P}{\Delta |V|}
\]

\[
\Delta |V| = -\left[ B'' \right]^{-1} \frac{\Delta Q}{\Delta |V|}
\]
\[ \delta_i^{k+1} = \delta_i^k + \Delta \delta^k \]
\[ |V_i^{k+1}| = |V_i^k| + \Delta |V_i^k| \]

- This method requires more iterations than NR method but less time per iteration
- It is useful for in contingency analysis
COMPARISION BETWEEN ITERATIVE METHODS

Gauss – Seidal Method

1. Computer memory requirement is less.
2. Computation time per iteration is less.
3. It requires less number of arithmetic operations to complete an iteration and ease in programming.
4. No. of iterations are more for convergence and rate of convergence is slow (linear convergence characteristic.
5. No. of iterations increases with the increase of no. of buses.
NEWTON – RAPHSON METHOD

- Superior convergence because of quadratic convergence.
- It has an 1:8 iteration ratio compared to GS method.
- More accurate.
- Smaller no. of iterations and used for large size systems.
- It is faster and no. of iterations is independent of the no. of buses.
- Technique is difficult and calculations involved in each iteration are more and thus computation time per iteration is large.
- Computer memory requirement is large, as the elements of jacobian matrix are to be computed in each iteration.
- Programming logic is more complex.
UNIT III

FAULT ANALYSIS-BALANCED FAULT
Need for fault analysis

- To determine the magnitude of fault current throughout the power system after fault occurs.
- To select the ratings for fuses, breakers and switchgear.
- To check the MVA ratings of the existing circuit breakers when new generators are added into a system.
BALANCED THREE PHASE FAULT

- All the three phases are short circuited to each other and to earth.
- Voltages and currents of the system balanced after the symmetrical fault occurred. It is enough to consider any one phase for analysis.

SHORT CIRCUIT CAPACITY

- It is the product of magnitudes of the prefault voltage and the post fault current.
- It is used to determine the dimension of a bus bar and the interrupting capacity of a circuit breaker.
Short Circuit Capacity (SCC)

\[ |SCC| = |V^0||I_F| \]

\[ |I_F| = \frac{|V_T|}{|Z_T|} \]

\[ |SCC|_{1\phi} = \frac{|V_T|^2}{|Z_T|} = \frac{S_{b,1\phi}}{|Z_T|_{p.u}} \text{ MVA} / \phi \]

\[ |SCC|_{3\phi} = \frac{S_{b,3\phi}}{|Z_T|_{p.u}} \text{ MVA} \]

\[ I_f = \frac{|SCC|_{3\phi} \times 10^6}{\sqrt{3} \times V_{L,b} \times 10^6} \]
Procedure for calculating short circuit capacity and fault current

- Draw a single line diagram and select common base $S_b$ MVA and kV
- Draw the reactance diagram and calculate the total p.u impedance from the fault point to source (Thevenin impedance $Z_T$)
- Determine SCC and $I_f$
ALGORITHM FOR SHORT CIRCUIT ANALYSIS USING BUS IMPEDANCE MATRIX

• Consider a n bus network. Assume that three phase fault is applied at bus k through a fault impedance \( z_f \).

• Prefault voltages at all the buses are

\[
V_{bus} (0) = \begin{bmatrix}
V_1(0) \\
V_2(0) \\
\vdots \\
V_k(0) \\
\vdots \\
V_n(0)
\end{bmatrix}
\]

• Draw the Thevenin equivalent circuit i.e Zeroing all voltage sources and add voltage source \( V_k(0) \) at faulted bus k and draw the reactance diagram.
• The change in bus voltage due to fault is

\[
\Delta V_{bus} = \begin{bmatrix}
\Delta V_1 \\
. \\
. \\
. \\
\Delta V_k \\
. \\
\Delta V_n
\end{bmatrix}
\]

• The bus voltages during the fault is

\[
V_{bus}(F) = V_{bus}(0) + \Delta V_{bus}
\]

• The current entering into all the buses is zero. The current entering into faulted bus \(k\) is \(-ve\) of the current leaving the bus \(k\)
\[ \Delta V_{\text{bus}} = Z_{\text{bus}} I_{\text{bus}} \]

\[
\begin{pmatrix}
Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kn} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
Z_{n1} & \cdots & Z_{nk} & \cdots & Z_{nn}
\end{pmatrix}
\begin{bmatrix}
0 \\
\vdots \\
-1_k(F) \\
\vdots \\
0
\end{bmatrix}
\]

\[ V_k(F) = V_k(0) - Z_{kk} I_k(F) \]

\[ V_f(F) = Z_f I_k(F) \]

\[ I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f} \]

\[ V_i(F) = V_i(0) - Z_{ik} I_k(F) \]
UNIT IV

FAULT ANALYSIS – UNBALANCED FAULTS
INTRODUCTION

UNSYMMETRICAL FAULTS

- One or two phases are involved
- Voltages and currents become unbalanced and each phase is to be treated individually
- The various types of faults are
  - Shunt type faults
    1. Line to Ground fault (LG)
    2. Line to Line fault (LL)
    3. Line to Line to Ground fault (LLG)
  - Series type faults
    Open conductor fault (one or two conductor open fault)
FUNDAMENTALS OF SYMMETRICAL COMPONENTS

- Symmetrical components can be used to transform three phase unbalanced voltages and currents to balanced voltages and currents
- Three phase unbalanced phasors can be resolved into following three sequences
  1. Positive sequence components
  2. Negative sequence components
  3. Zero sequence components
Positive sequence components

Three phasors with equal magnitudes, equally displaced from one another by 120° and phase sequence is same as that of original phasors.

\[ V_{a1}, V_{b1}, V_{c1} \]

Negative sequence components

Three phasors with equal magnitudes, equally displaced from one another by 120° and phase sequence is opposite to that of original phasors.

\[ V_{a2}, V_{b2}, V_{c2} \]

Zero sequence components

Three phasors with equal magnitudes and displaced from one another by 0°

\[ V_{a0}, V_{b0}, V_{c0} \]
RELATIONSHIP BETWEEN UNBALANCED VECTORS AND SYMMETRICAL COMPONENTS

\[ V_a = V_{a0} + V_{a1} + V_{a2} \]
\[ V_b = V_{b0} + V_{b1} + V_{b2} \]
\[ V_c = V_{c0} + V_{c1} + V_{c2} \]

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix}
\begin{bmatrix}
V_{a0} \\
V_{a1} \\
V_{a2}
\end{bmatrix}
\]

\[ A = \begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix} \]

Similarly we can obtain for currents also
SEQUENCE IMPEDANCE

Impedances offered by power system components to positive, negative and zero sequence currents.

Positive sequence impedance

The impedance of a component when positive sequence currents alone are flowing.

Negative sequence impedance

The impedance of a component when negative sequence currents alone are flowing.

Zero sequence impedance

The impedance of a component when zero sequence currents alone are flowing.
SEQUENCE NETWORK

SEQUENCE NETWORK FOR GENERATOR

positive sequence network

negative sequence network

Zero sequence network
SEQUENCE NETWORK FOR TRANSMISSION LINE

positive sequence network

negative sequence network

Zero sequence network
SEQUENCE NETWORK FOR TRANSFORMER

positive sequence network  negative sequence network  Zero sequence network
SEQUENCE NETWORK FOR LOAD

positive sequence network  negative sequence network  Zero sequence network
Consider a fault between phase a and ground through an impedance $z_f$

\[ I_b = 0 \]
\[ I_c = 0 \]
\[ V_a = z_f I_a \]
\[ I_{a1} = I_{a2} = I_{a0} = I_a / 3 \]
\[ I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0 + 3Z_f} \]
Consider a fault between phase b and c through an impedance $z_f$

$$I_a = 0$$
$$I_c = -I_b$$
$$V_b - V_c = I_b Z^f$$
$$I_{a2} = -I_{a1}$$
$$I_{a0} = 0$$
$$V_{a1} - V_{a2} = Z_f I_{a1}$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + 3Z^f}$$

$$I_b = -I_c = \frac{-jE_a}{Z_1 + Z_2 + 3Z^f}$$
Consider a fault between phase b and c through an impedance $z_f$ to ground.
SINGLE LINE TO GROUND FAULT USING $Z_{bus}$

Consider a fault between phase a and ground through an impedance $z_f$ at bus k

For a fault at bus k the symmetrical components of fault current

$$I_k^0 = I_k^1 = I_k^2 = \frac{V_k(0)}{Z_{kk}^1 + Z_{kk}^2 + Z_{kk}^0 + 3Z_f}$$

Where $Z_{kk}^1$, $Z_{kk}^2$, $Z_{kk}^0$ are the diagonal elements in the k axis of the $Z_{bus}$ & $V_k(0)$ is the prefault voltage at bus k.

Fault phase current $I_k^{abc} = A I_k^{012}$
LINE TO LINE (LL) FAULT

Consider a fault between phase b and c through an impedance $z_f$

\[ I_k^0 = 0 \]
\[ I_k^1 = -I_k^2 = \frac{V_k(0)}{Z_{kk}^1 + Z_{kk}^2 + Z^f} \]
DOUBLE LINE TO GROUND (LLG) FAULT

Consider a fault between phase b and c through an impedance $z_f$ to ground.

Bus k of network

$$I_k^1 = \frac{V_k(0)}{Z_{kk}^1 + \frac{Z_{kk}^2}{Z_{kk} + 3Z_f}}$$

$$I_k^2 = -\frac{V_k(0) - Z_{kk}I_k^1}{Z_{kk}^2}$$

$$I_k^0 = -\frac{V_k(0) - Z_{kk}I_k^1}{Z_{kk}^0 + 3Z_f}$$

$$I_k(F) = I_k^b + I_k^c$$
BUS VOLTAGES AND LINE CURRENTS DURING FAULT

\[ V_{i0}^0 (F) = 0 - Z_{ik}^0 I_k^0 \]
\[ V_{i1}^1 (F) = V_{i0}^0 (0) - Z_{ik}^1 I_k^1 \]
\[ V_{i2}^2 (F) = 0 - Z_{ik}^2 I_k^2 \]

\[ I_{ij}^0 = \frac{V_{i0}^0 (F) - V_{j0}^0 (F)}{Z_{ij}^0} \]
\[ I_{ij}^1 = \frac{V_{i1}^1 (F) - V_{j1}^1 (F)}{Z_{ij}^1} \]
\[ I_{ij}^2 = \frac{V_{i2}^2 (F) - V_{j2}^2 (F)}{Z_{ij}^2} \]
UNIT V

STABILITY ANALYSIS
STABILITY

- The tendency of a power system to develop restoring forces equal to or greater than the disturbing forces to maintain the state of equilibrium.
- Ability to keep the machines in synchronism with another machine
CLASSIFICATION OF STABILITY

- Steady state stability
  Ability of the power system to regain synchronism after small and slow disturbances (like gradual power changes)

- Dynamic stability
  Ability of the power system to regain synchronism after small disturbances occurring for a long time (like changes in turbine speed, change in load)

- Transient stability
  This concern with sudden and large changes in the network conditions i.e. sudden changes in application or removal of loads, line switching operating operations, line faults, or loss of excitation.
Steady state limit is the maximum power that can be transferred without the system becoming unstable when the load is increased gradually under steady state conditions.

Transient limit is the maximum power that can be transferred without the system becoming unstable when a sudden or large disturbance occurs.
Swing Equation for Single Machine Infinite Bus System

• The equation governing the motion of the rotor of a synchronous machine

\[ J \frac{d^2 \theta_m}{dt^2} = T_a = T_m - T_e \]

where

- \( J = \) The total moment of inertia of the rotor (kg-m\(^2\))
- \( \theta_m = \) Singular displacement of the rotor
- \( T_m = \) Mechanical torque (N-m)
- \( T_e = \) Net electrical torque (N-m)
- \( T_a = \) Net accelerating torque (N-m)
\[
\theta_m = \omega_{sm} t + \delta_m
\]
\[
\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}
\]
\[
\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}
\]
\[
J \omega_m \frac{d^2\delta_m}{dt^2} = p_a = p_m - p_e
\]

- Where \( p_m \) is the shaft power input to the machine
- \( p_e \) is the electrical power
- \( p_a \) is the accelerating power
\[ J \omega_m = M \]
\[ M \frac{d^2 \delta_m}{dt^2} = p_a = p_m - p_e \]
\[ M = \frac{2H}{\omega_{sm}} S_{\text{machine}} \]
\[ \frac{2H}{\omega_{sm}} \frac{d^2 \delta_m}{dt^2} = \frac{p_a}{S_{\text{machine}}} = \frac{p_m - p_e}{S_{\text{machine}}} \]
\[ \frac{2H}{\omega_s} \frac{d^2 \delta}{dt^2} = p_a = p_m - p_e \]
\[ \omega_s = 2\pi f \]
\[ \frac{H}{\pi f_0} \frac{d^2 \delta}{dt^2} = p_a = \left(p_m\right) - p_e \]
\[ \frac{d^2 \delta}{dt^2} = \frac{\pi f_0}{H} \left(p_m - p_{2\text{max}} \sin \delta\right) = \frac{\pi f_0}{H} p_a \]
\[ \frac{d \delta}{dt} = \Delta \omega \]
\[ \frac{d \Delta \omega}{dt} = \frac{\pi f_0}{H} p_a = \frac{d^2 \delta}{dt^2} \quad \text{p.u} \]

H=machine inertia constant

\[ \delta \text{ and } \omega_s \text{ are in electrical radian} \]
Swing Equation for Multimachine System

\[ S_{machine} = \text{machine rating(base)} \]

\[ S_{system} = \text{system base} \]

\[ \frac{H_{system}}{\pi f} \frac{d^2 \delta}{dt^2} = p_a = p_m - p_e \quad \text{p.u} \]

\[ H_{system} = H_{machine} \frac{S_{machine}}{S_{system}} \]
Rotor Angle Stability

- It is the ability of interconnected synchronous machines of a power system to maintain in synchronism. The stability problem involves the study of the electro mechanical oscillations inherent in power system.

- Types of Rotor Angle Stability
  1. Small Signal Stability (or) Steady State Stability
  2. Transient stability
Voltage Stability

- It is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance.
- The major factor for instability is the inability of the power system to meet the demand for reactive power.
• **Mid Term Stability**
  It represents transition between short term and long term responses.
  Typical ranges of time periods.
  1. Short term : 0 to 10s
  2. Mid Term : 10 to few minutes
  3. Long Term : a few minutes to 10’s of minutes

• **Long Term Stability**
  Usually these problem be associated with
  1. Inadequacies in equipment responses.
  2. Poor co-ordination of control and protection equipment.
  3. Insufficient active/reactive power reserves.
Equal Area Criterion

• This is a simple graphical method to predict the transient stability of two machine system or a single machine against infinite bus. This criterion does not require swing equation or solution of swing equation to determine the stability condition.

• The stability conditions are determined by equating the areas of segments on power angle diagram.
Power-angle curve for equal area criterion

Multiplying swing equation by \(d\delta/dt\) on both sides

\[
\frac{H}{\omega_0} \left( \frac{d\delta}{dt} \right)^2 = (P_m - P_e) \frac{d\delta}{dt}
\]

\[
\frac{d}{dt} \left( \frac{d\delta}{dt} \right)^2 = 2 \left( \frac{d\delta}{dt} \right) \left( \frac{d^2\delta}{dt^2} \right)
\]

Multiplying both sides of the above equation by \(dt\) and then integrating between two arbitrary angles \(\delta_0\) and \(\delta_c\)
Once a fault occurs, the machine starts accelerating. Once the fault is cleared, the machine keeps on accelerating before it reaches its peak at $\delta_c$.

The area of accelerating $A_1$

$$A_1 = \int_{\delta_0}^{\delta_c} (P_m - P_e) d\delta = 0$$

The area of deceleration is given by $A_2$

$$A_2 = \int_{\delta_c}^{\delta} (P_e - P_m) d\delta = 0$$

If the two areas are equal, i.e., $A_1 = A_2$, then the power system will be stable.
Critical Clearing Angle ($\delta_{cr}$) maximum allowable value of the clearing time and angle for the system to remain stable are known as critical clearing time and angle.

$\delta_{cr}$ expression can be obtained by substituting $\delta_{c} = \delta_{cr}$ in the equation $A_1 = A_2$

\[
\int_{\delta_0}^{\delta_{cr}} (P_m - P_e) d\delta = \int_{\delta_{cr}}^{\delta_{cr}} (P_e - P_m) d\delta
\]

Substituting $P_e = 0$ in swing equation

\[
\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}P_m
\]

Integrating the above equation

\[
\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H}P_m dt = \frac{\omega_s}{2H}P_m t
\]
Replacing $\delta$ by $\delta_{cr}$ and $t$ by $t_{cr}$ in the above equation, we get the critical clearing time as

$$
\delta = \int_{0}^{t} \frac{m_s}{2H} P_m t \, dt = \frac{m_s}{4H} P_m t^2 + \delta_0
$$

$$
t_{cr} = \sqrt{\frac{4H}{m_s P_m} (\delta_{cr} - \delta_0)}
$$
Factors Affecting Transient Stability

• Strength of the transmission network within the system and of the tie lines to adjacent systems.
• The characteristics of generating units including inertia of rotating parts and electrical properties such as transient reactance and magnetic saturation characteristics of the stator and rotor.
• Speed with which the faulted lines or equipments can be disconnected.
Numerical Integration methods

- Modified Euler’s method
- Runge-Kutta method
MODIFIED EULER’S METHOD

• Using first derivative of the initial point next point is obtained
  \[ X_1^p = X_0 + \frac{dX}{dt} \Delta t \]
  the step \[ t_1 = t_0 + \Delta t \]

• Using this \( x_1^p \) \( \frac{dx}{dt} \) at \( x_1^p = f(t_1, x_1^p) \)

• Corrected value is

\[
X_{i+1}^c = X_i + \left( \frac{dx}{dt} \right)_{x_i} + \left( \frac{dx}{dt} \right)_{x_i^p} \Delta t
\]

\[
X_{i+1}^c = X_i + \left( \frac{dx}{dt} \right)_{x_i} + \left( \frac{dx}{dt} \right)_{x_i^p} \Delta t
\]
Numerical Solution of the swing equation

- Input power $p_m=$constant
- At steady state $p_e=p_m$,
  \[ \delta_0 = \sin^{-1}\left(\frac{p_m}{p_{1\text{max}}}\right) \]
  \[ p_{1\text{max}} = \frac{|E^*||V|}{X_1} \]
- At synchronous speed
  \[ \Delta \omega_0 = 0 \]
  \[ p_{2\text{max}} = \frac{|E^*||V|}{X_2} \]
The swing equation

\[
\frac{H}{\pi f_0} \frac{d^2 \delta}{dt^2} = p_a = (p_m) - p_e
\]

\[
\frac{d^2 \delta}{dt^2} = \frac{\pi f_0}{H} (p_m - p_{2\max} \sin \delta) = \frac{\pi f_0}{H} p_a
\]

\[
\frac{d \delta}{dt} = \Delta \omega
\]

\[
\frac{d \Delta \omega}{dt} = \frac{\pi f_0}{H} p_a = \frac{d^2 \delta}{dt^2}
\]

Applying Modified Eulers method to above equation

\[
t_1 = t_0 + \Delta t
\]

\[
\delta^p_{i+1} = \delta_i + \left( \frac{d \delta}{dt} \right)_{\Delta \omega_i} \Delta t
\]

\[
\Delta \omega^p_{i+1} = \Delta \omega_i + \left( \frac{d \Delta \omega}{dt} \right)_{\delta_i} \Delta t
\]
• The derivatives at the end of interval

\[
\left( \frac{d\delta}{dt} \right)_{\Delta\omega_{i+1}^p} = \Delta\omega_{i+1}^p
\]

\[
\left( \frac{d\Delta\omega}{dt} \right)_{\delta_{i+1}^p} = \left( \frac{\pi f_0}{H} p_a \right)_{\delta_{i+1}^p}
\]

The corrected value

\[
\delta_{i+1}^c = \delta_i + \left( \frac{d\delta}{dt} \right)_{\Delta\omega_i} + \left( \frac{d\delta}{dt} \right)_{\Delta\omega_{i+1}^p} \Delta t
\]

\[
\Delta\omega_{i+1}^c = \Delta\omega_i + \left( \frac{d\Delta\omega}{dt} \right)_{\delta_i} + \left( \frac{d\Delta\omega}{dt} \right)_{\delta_{i+1}^p} \Delta t
\]
Runge-Kutta Method

• Obtain a load flow solution for pretransient conditions
• Calculate the generator internal voltages behind transient reactance.
• Assume the occurrence of a fault and calculate the reduced admittance matrix
• Initialize time count $K=0, J=0$
• Determine the eight constants

$$K_1^k = f_1(\delta^k, \omega^k) \Delta t$$
$$l_1^k = f_2(\delta^k, \omega^k) \Delta t$$

$$K_2^k = f_1(\delta^k + \frac{K_1^k}{2}, \omega^k + \frac{l_1^k}{2}) \Delta t$$
$$l_2^k = f_2(\delta^k + \frac{K_1^k}{2}, \omega^k + \frac{l_1^k}{2}) \Delta t$$

$$K_3^k = f_1(\delta^k + \frac{K_2^k}{2}, \omega^k + \frac{l_2^k}{2}) \Delta t$$
$$l_3^k = f_2(\delta^k + \frac{K_2^k}{2}, \omega^k + \frac{l_2^k}{2}) \Delta t$$

$$K_4^k = f_1(\delta^k + \frac{K_3^k}{2}, \omega^k + \frac{l_3^k}{2}) \Delta t$$
$$l_4^k = f_2(\delta^k + \frac{K_3^k}{2}, \omega^k + \frac{l_3^k}{2}) \Delta t$$

$$\Delta \delta^k = \frac{(K_1^k + 2K_2^k + 2K_3^k + K_4^k)}{6}$$

$$\Delta \omega^k = \frac{(l_1^k + 2l_2^k + 2l_3^k + l_4^k)}{6}$$
• Compute the change in state vector

\[
\Delta \delta^k = \frac{\left( K_1^k + 2K_2^k + 2K_3^k + K_4^k \right)}{6}
\]

\[
\Delta \omega^k = \frac{\left( l_1^k + 2l_2^k + 2l_3^k + l_4^k \right)}{6}
\]

• Evaluate the new state vector

\[
\delta^{k+1} = \delta^k + \Delta \delta^k
\]

\[
\omega^{k+1} = \omega^k + \Delta \omega^k
\]

• Evaluate the internal voltage behind transient reactance using the relation

\[
E_{p}^{k+1} = |E_p^k| \cos \delta_{p}^{k+1} + j |E_p^k| \sin \delta_{p}^{k+1}
\]

• Check if \( t < t_c \) yes \( K = K + 1 \)
• Check if \( j = 0 \), yes modify the network data and obtain the new reduced admittance matrix and set \( j = j + 1 \)
• set \( K = K + 1 \)
• Check if \( K < K_{\text{max}} \), yes start from finding 8 constants