Analysis of Unbalanced Systems

- Except for the balanced three-phase fault, faults result in an unbalanced system.
- The most common types of faults are single line-ground (SLG) and line-line (LL). Other types are double line-ground (DLG), open conductor, and balanced three phase.
- The easiest method to analyze unbalanced system operation due to faults is through the use of symmetrical components.
Symmetrical Components

- The key idea of symmetrical component analysis is to decompose the unbalanced system into three sequence of balanced networks. The networks are then coupled only at the point of the unbalance (i.e., the fault).

- The three sequence networks are known as the:
  - positive sequence (this is the one we’ve been using)
  - negative sequence
  - zero sequence
Symmetrical Components

Unsymmetrical Fault

Unbalance System

Unbalance Currents

$I_C$

$I_A$

$I_B$

Symmetrical components

Balance Systems

Zero Sequence

Positive Sequence

Negative Sequence

Three balanced Systems

Sequence Currents

zero sequence

positive sequence

negative sequence

AI

BI

CI
Assuming three unbalance voltage phasors, $V_A$, $V_B$ and $V_C$ having a positive sequence ($abc$). Using symmetrical components it is possible to represent each phasor voltage as:

$$
V_A = V_A^0 + V_A^+ + V_A^-
$$

$$
V_B = V_B^0 + V_B^+ + V_B^-
$$

$$
V_C = V_C^0 + V_C^+ + V_C^-
$$

Where the symmetrical components are:
Symmetrical Components

The Positive Sequence Components \(( V_A^+, V_B^+, V_C^+ \) )
Three phasors
Equal in magnitude
Displaced by 120° in phase
Having the same sequence as the original phasors (abc)

The Negative Sequence Components \(( V_A^-, V_B^-, V_C^- \) )
Three phasors
Equal in magnitude
Displaced by 120° in phase
Having the opposite sequence as the original phasors (acb)

The zero Sequence Components \(( V_A^0, V_B^0, V_C^0 \) )
Three phasors
Equal in magnitude
Having the same phase shift (in phase)
**Example**

### Zero Sequence
- \( V_{A0} \)
- \( V_{B0} \)
- \( V_{C0} \)

### Positive Sequence
\[ V_A = V_A^0 + V_A^+ + V_A^- \]
\[ V_B = V_B^0 + V_B^+ + V_B^- \]
\[ V_C = V_C^0 + V_C^+ + V_C^- \]

### Negative Sequence

### Synthesis Unsymmetrical phasors using symmetrical components

**Unbalance Voltage**
Sequence Set Representation

Any arbitrary set of three phasors, say $I_a$, $I_b$, $I_c$ can be represented as a sum of the three sequence sets:

$$I_a = I_a^0 + I_a^+ + I_a^-$$
$$I_b = I_b^0 + I_b^+ + I_b^-$$
$$I_c = I_c^0 + I_c^+ + I_c^-$$

where

$I_a^0, I_b^0, I_c^0$ is the zero sequence set

$I_a^+, I_b^+, I_c^+$ is the positive sequence set

$I_a^-, I_b^-, I_c^-$ is the negative sequence set
Conversion Sequence to Phase

Only three of the sequence values are unique, $I_a^0, I_a^+, I_a^-$; the others are determined as follows:

\[ \alpha = 1 \angle 120^\circ \quad \alpha + \alpha^2 + \alpha^3 = 0 \quad \alpha^3 = 1 \]

\[ I_a^0 = I_b^0 = I_c^0 \quad \text{(since by definition they are all equal)} \]

\[ I_b^+ = \alpha^2 I_a^+ \quad I_c^+ = \alpha I_a^+ \quad I_b^- = \alpha I_a^- \quad I_c^+ = \alpha^2 I_a^- \]

\[
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix} =
\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} +
\begin{bmatrix} 1 \\ \alpha^2 \\ \alpha \end{bmatrix} I_a^+ +
\begin{bmatrix} 1 \\ \alpha^2 \\ \alpha \end{bmatrix} I_a^- =
\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}
\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}
\]
Define the symmetrical components transformation matrix

\[ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \]

Then \[ I = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = A \begin{bmatrix} I^0_a \\ I^+_a \\ I^-_a \end{bmatrix} = A \begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix} = A I_s \]
By taking the inverse we can convert from the phase values to the sequence values

\[ I_s = A^{-1}I \]

with

\[ A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \]

Sequence sets can be used with voltages as well as with currents
Example

If the values of the fault currents in a three phase system are:

\[ I_A = 150 \angle 45 \quad I_B = 250 \angle 150 \quad I_C = 100 \angle 300 \]

Find the symmetrical components?

**Solution:**

\[ V_+ = \frac{1}{3}(V_A + \alpha V_B + \alpha^2 V_C) \]
\[ V_- = \frac{1}{3}(V_A + \alpha^2 V_B + \alpha V_C) \]
\[ V_0 = \frac{1}{3}(V_A + V_B + V_C) \]

\[ I_+ = \frac{1}{3}(I_A + \alpha I_B + \alpha^2 I_C) = \frac{1}{3}(150 \angle 45^\circ + 250 \angle 270^\circ + 100 \angle 180^\circ) = 48.02 \angle -87.6^\circ \]

\[ I_- = \frac{1}{3}(I_A + \alpha^2 I_B + \alpha I_C) = 163.21 \angle 40.45^\circ \]

\[ I_0 = \frac{1}{3}(I_A + I_B + I_C) = \frac{1}{3}(106.04 + j106.07 + j106.07 - 216.51 + j125.00 + 50 - j86.6) = 52.2 \angle 112.7^\circ \]
Example

If the values of the sequence voltages in a three phase system are:

\[ V_o = 100 \]
\[ V_+ = 200 \angle 60 \]
\[ V_- = 100 \angle 120 \]

Find the three phase voltages

Solution:

\[ V_A = 200 \angle 60 + 100 \angle 120 + 100 \]

\[ V_A = 300 \angle 60 \]

\[ V_B = 1 \angle 240(200 \angle 60) + 1 \angle 120(100 \angle 120) + 100 \]

\[ V_B = 300 \angle -60 \]

\[ V_C = 1 \angle 120(200 \angle 60) + 1 \angle 240(100 \angle 120) + 100 \]

\[ V_C = 0 \]
Consider the following Y-connected load:

\[ I_n = I_a + I_b + I_c \]

\[ V_{ag} = I_a Z_y + I_n Z_n \]

\[ V_{bg} = (Z_Y + Z_n) I_a + Z_n I_b + Z_n I_c \]

\[ V_{cg} = Z_n I_a + Z_n I_b + (Z_Y + Z_n) I_c \]

\[
\begin{bmatrix}
V_{ag} \\
V_{bg} \\
V_{cg}
\end{bmatrix} =
\begin{bmatrix}
Z_y + Z_n & Z_n & Z_n \\
Z_n & Z_y + Z_n & Z_n \\
Z_n & Z_n & Z_y + Z_n
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]
Use of Symmetrical Components

\[
\begin{bmatrix}
V_{ag} \\
V_{bg} \\
V_{cg}
\end{bmatrix}
= 
\begin{bmatrix}
Z_y + Z_n & Z_n & Z_n \\
Z_n & Z_y + Z_n & Z_n \\
Z_n & Z_n & Z_y + Z_n
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

\[V = Z I, \quad V = A V_s, \quad I = A I_s\]

\[AV_s = Z A I_s \quad \rightarrow \quad V_s = A^{-1} Z A I_s\]

\[A^{-1} Z A = 
\begin{bmatrix}
Z_y + 3Z_n & 0 & 0 \\
0 & Z_y & 0 \\
0 & 0 & Z_y
\end{bmatrix}\]
Networks are Now Decoupled

\[
\begin{bmatrix}
V^0 \\
V^+ \\
V^-
\end{bmatrix} =
\begin{bmatrix}
Z_y + 3Z_n & 0 & 0 \\
0 & Z_y & 0 \\
0 & 0 & Z_y
\end{bmatrix}
\begin{bmatrix}
I^0 \\
I^+ \\
I^-
\end{bmatrix}
\]

Systems are decoupled

\[
V^0 = (Z_y + 3Z_n) I^0 \quad V^+ = Z_y I^+ \\
V^- = Z_y I^-
\]
Y-connected load (Isolated Neutral):

If the neutral point of a Y-connected load is not grounded, therefore, no zero sequence current can flow, and

\[ Z_n = \infty \]

Symmetrical circuits for Y-connected load with neutral point is not connected to ground are presented as shown:
The Delta circuit can not provide a path through neutral. Therefore for a *Delta connected load* or its *equivalent Y-connected* can not contain any zero sequence components.

\[ V_{ab} = Z_\Delta I_{ab}, \quad V_{bc} = Z_\Delta I_{bc}, \quad V_{ca} = Z_\Delta I_{ca} \]

The summation of the line-to-line voltages or phase currents are always zero

\[ \frac{1}{3} (V_{ab} + V_{bc} + V_{ca}) = V_{ab0} = 0 \quad \text{and} \quad \frac{1}{3} (I_{ab} + I_{bc} + I_{ca}) = I_{ab0} = 0 \]

Therefore, for a *Delta-connected loads* without sources or mutual coupling there will be no zero sequence currents at the lines (There are *some cases where a circulating currents may circulate inside a delta load and not seen at the terminals of the zero sequence circuit*).
Sequence diagrams for lines

- Similar to what we did for loads, we can develop sequence models for other power system devices, such as lines, transformers and generators.

- For transmission lines, assume we have the following, with mutual impedances.
Assume the phase relationships are

\[
\begin{bmatrix}
\Delta V_a \\
\Delta V_b \\
\Delta V_c
\end{bmatrix} =
\begin{bmatrix}
Z_s & Z_m & Z_m \\
Z_m & Z_s & Z_m \\
Z_m & Z_m & Z_s
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

where

\[Z_s = \text{self impedance of the phase}\]
\[Z_m = \text{mutual impedance between the phases}\]

Writing in matrix form we have

\[\Delta V = ZI\]
Sequence diagrams for lines, cont’d

Similar to what we did for the loads, we can convert these relationships to a sequence representation

\[
\Delta V = Z I \\
\Delta V = A \Delta V_s \\
I = A I_s
\]

\[
A \Delta V_s = Z A I_s 
\rightarrow 
\Delta V_s = A^{-1} Z A I_s
\]

\[
A^{-1} Z A = \begin{bmatrix}
Z_s + 2Z_m & 0 & 0 \\
0 & Z_s - Z_m & 0 \\
0 & 0 & Z_s - Z_m
\end{bmatrix}
\]
Sequence diagrams for lines, cont’d

Therefore,

\[ Z_0 = Z_s + 2Z_m \]
\[ Z_+ = Z_s - Z_m \]
\[ Z_- = Z_s - Z_m \]

Where,

\[ Z_s = Z_{aa} + Z_{nn} - 2Z_an \]
\[ Z_m = Z_{ab} + Z_{nn} - 2Z_an \]

The ground wires \((\text{above overhead TL})\) combined with the earth works as a neutral conductor with impedance parameters that effects the zero sequence components. Having a good grounding (depends on the soil resistively), then the voltages to the neutral can be considered as the voltages to ground.
Sequence diagrams for generators

- Key point: generators only produce positive sequence voltages; therefore only the positive sequence has a voltage source.

During a fault $Z^+ \approx Z^- \approx X_d$. The zero sequence impedance is usually substantially smaller. The value of $Z_n$ depends on whether the generator is grounded.
Sequence diagrams for Transformers

- The positive and negative sequence diagrams for transformers are similar to those for transmission lines.

- The zero sequence network depends upon both how the transformer is grounded and its type of connection. The easiest to understand is a double grounded wye-wye.
Transformer Sequence Diagrams